

In[1]:= $S = \text{Cot}[fov / 2];$

In[2]:= $T = -S * (\text{width} / \text{height});$

In[3]:=
$$P = \begin{pmatrix} S & 0 & 0 & 0 \\ 0 & T & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix};$$

In[4]:= $\text{PerspectiveDivide}[\{x_, y_, z_, w_\}] := \{x, y, z\} / w$

In[5]:= $\text{GLViewportTransform}[\{x_, y_, z_\}] := \left\{ \frac{\text{width}}{2} (x + 1), \frac{\text{height}}{2} (y + 1) \right\}$

In[6]:= $\text{GLViewportTransform}[\text{PerspectiveDivide}[P.\{x, y, z, 1\}]]$

Out[6]=
$$\left\{ \frac{1}{2} \text{width} \left(1 - \frac{x \text{Cot}\left[\frac{fov}{2}\right]}{z} \right), \frac{1}{2} \text{height} \left(1 + \frac{\text{width } y \text{Cot}\left[\frac{fov}{2}\right]}{\text{height } z} \right) \right\}$$

In[7]:= $\text{Expand}[\%]$

Out[7]=
$$\left\{ \frac{\text{width}}{2} - \frac{\text{width } x \text{Cot}\left[\frac{fov}{2}\right]}{2 z}, \frac{\text{height}}{2} + \frac{\text{width } y \text{Cot}\left[\frac{fov}{2}\right]}{2 z} \right\}$$

In[8]:= $\text{MythProject}[\{x_, y_, z_\}] := \left\{ \frac{\text{width}}{2} - \frac{x}{z} * \frac{(\text{width}/2)}{\text{Tan}[fov/2]}, \frac{\text{height}}{2} + \frac{y}{z} * \frac{(\text{width}/2)}{\text{Tan}[fov/2]} \right\}$

In[9]:= $\text{MythProject}[\{x, y, z\}]$

Out[9]=
$$\left\{ \frac{\text{width}}{2} - \frac{\text{width } x \text{Cot}\left[\frac{fov}{2}\right]}{2 z}, \frac{\text{height}}{2} + \frac{\text{width } y \text{Cot}\left[\frac{fov}{2}\right]}{2 z} \right\}$$

In[10]:= $\text{Reduce}[\text{MythProject}[\{x, y, z\}] == \text{GLViewportTransform}[\text{PerspectiveDivide}[P.\{x, y, z, 1\}]]]$

Out[10]= True